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A Theoretical Study on Modeling the Nonlinear Dynamics of a Knee Emulator

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Abstract

Dynamic knee emulators are used by orthopedic surgeons and design engineers to develop and test knee prostheses. This paper presents the development of a theoretical model for a dynamic knee emulator with the aim of increasing the performance of this test rig. The test rig is designed to perform a squat movement (i.e. bending of the knees). A theoretical nonlinear model is necessary to design a control strategy. Based on Lagrangian dynamics, a nonlinear model for flexing motion of the knee rig is derived in this work.

1 Introduction

Nonlinear dynamic knee emulating machines aim to reproduce the natural movement of a human knee in a controlled environment [1]. The nonlinear dynamic knee emulator, or knee rig, is used to reproduce the bending movement of the knees i.e. knee flexion-extension such as occurs in situations of riding a bicycle, climbing stairs, sitting down or rising from a chair [2]. The need of these in-vitro machines is supported by the increasing development of new knee implants and new surgical techniques for which pre-clinical investigations are a prerequisite.The performance of the total knee replacement (TKR) can be evaluated under well-controlled loading conditions, allowing a comparison between different types of TKR techniques. These in-vitro experiments also give a better understanding of the biomechanics of the knee since post-mortem human joint specimens can be used to perform measurements which are otherwise impossible in an in-vivo setup. Experiments on the knee rig also provide insight into different kinds of abnormalities in the knee joint, e.g. bone

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deformations [3], abnormal force distributions [4] and changes to ligaments [5]. These abnormalities in the knee joint can have a big impact on the mobility of the patient and, consequently, on the quality of life.

Knee injuries account for 15 to 50% of all sports injuries [6]. Combine this fact with an aging population and the increased amount of knee injuries becomes evident. As a result, the amount of total knee replacements performed in the United States has doubled in the last two decades to an annual amount of 720000 procedures in 2010 costing \$ 15000 per procedure [7–9]. The hospital costs associated with total knee replacements have a total amount of \$ 14.3 billion each year [10]. Similar statistics are mentioned for the EU by the OECD [11].

The knee rig used in this research is based on the Oxford Rig [2] which was designed to perform a knee flexion-extension movement of a post-mortem human knee joint. In literature, two basic types of knee emulating machines have been described. The first type is a machine where an external force is applied which is balanced by the force in the quadriceps muscle while the second type is a machine where forces and motions are directly applied to the upper leg (i.e. the femur) and the lower leg (i.e. the tibia) [12]. The Oxford knee rig belongs to the first type with two main components: an ankle assembly and a hip assembly where the hip can move vertically up and down. The externally applied force is a fixed weight applied at the hip which is balanced by the quadriceps force [13]. The advantage of a type 1 machine is that no constraints are imposed on the movement of the bones as is the case for type 2 machines resulting in the natural 6 degrees-of-freedom of the human knee joint [12].

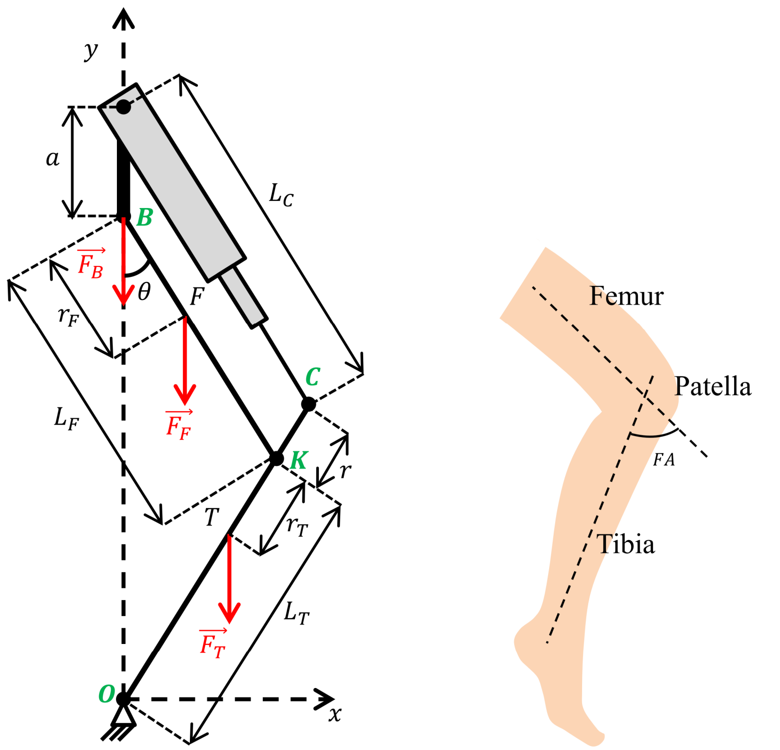
The control of a nonlinear dynamic knee emulator is currently done by either visual control of the person handling the machine, either closed loop control with the quadriceps force as controlled variable [14]. However, the longterm objective of our study is to develop a control strategy with the flexion angle as the controlled variable. This makes the system more challenging for control as the system's input and output are multiplied in the equations of motion that characterize the system. In order to design a control strategy for the system, a mathematical model of the system is necessary.

The structure of this paper is as follows: Section 2 discusses the mathematical model of the dynamic knee emulator using Lagrange theory. Section 3 shows simulations of the obtained model. Afterwards a conclusion section is given.

2 Mathematical Model

2.1 Description of the system

The schematic representation of the dynamic knee emulator is shown in figure 1a. As the motion in the sagittal plane (i.e. the plane that divides the human body in left and right) is dominant over the motions in the coronal and transverse plane (i.e. respectively the plane that divides the human body in front and back and plane that divides the human body in top and bottom), the knee rig is modeled as a 2D motion in the sagittal plane. The model consists of two segments representing the tibia (i.e. shank) and the femur (i.e. thigh). The patella (i.e. kneecap) is modeled as an extension of the tibia where the length of extension represents the height of the patella. The total body weight is taken into account as a concentrated mass in point $\boldsymbol{B}$. We assume that the segments of the femur and the tibia are homogeneous causing the gravitational forces to act on the center of mass (COM) of each segment. The ankle is a fixed point in space where we will place the origin of



(a) Schematic representation of the dynamic knee emulator.

(b) Definition of the flexion angle of the knee.

Fig. 1 Schematic representations.

our coordinate system.

The model includes also a linear actuator which in the flexion phase will extend with a velocity $v(t)$ of 10 mm/s and in the extension phase will decrease the length L_C with a velocity $v(t)$ of 10 mm/s. This velocity is the input of the system. The moving part of the linear actuator is connected with the patella (point C) by a cable. The objective is to be able to use the flexion angle of the knee as a controlled variable. The flexion angle of a knee joint is defined as the exterior angle between the tibia and the extension of the femur (see figure 1b). It is not possible to measure this angle FA in a post-mortem specimen without invasive procedures. However, the angle θ in figure 1a can be measured as a function of time using a potentiometer and the angle FA and the angle θ can be related to each other using the geometry of the system by the following equation:

$$FA = \frac{\pi}{2} + \theta - acos\left(\frac{L_F}{L_T} sin\theta\right) \tag{1}$$

Hence, the angle θ is chosen to be the output of the system and the measured value of θ will be used as feedback in the control strategy. The model parameters are taken for an average person from literature [15] and are defined as follows:

- θ = angle between the vertical direction and the femur
- L_F = length of the femur (0,41 m)

- L_T = length of the tibia (0,435 m)
- r = height of the patella (0,01 m)
- r_T = distance between center of mass of the tibia and the knee joint axis K (0,188 m)
- r_F = distance between center of mass of the femur and the hip joint axis B (0,178 m)
- m_T = mass of the tibia (3,72 kg)
- m_F = mass of the femur (8 kg)
- m_B = mass of the body (80 kg)
- I_T = inertia of the tibia around the ankle joint (0,291 kgm^2)
- I_F = inertia of the femur around COM (0,140 kgm^2)
- g = gravitational constant (9,81 N/kg)
- a = length between point B and the rotation point of the linear actuator (0,015 m)
- L_C = length between the patella (point C) and the rotation point of the linear actuator

2.2 Equation of motion using Lagrange theory

In the system described in previous section, θ is the only independent parameter that defines the complete configuration of the system. Therefore, the system has only one degree of freedom and only one equation of motion. The generalized equation of motion using the Lagrange theory [16] can be expressed as:

$$Q = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} \tag{2}$$

with q the generalized coordinate, Q the generalized force, t the time and L the Lagrangian of the system. In this case, we choose θ as the generalized coordinate q.

The Lagrangian is defined as:

$$L = T - U \tag{3}$$

with T the total kinetic energy of the system and U the total potential energy of the system.

The total kinetic energy of the system is given by:

$$T = \frac{1}{2}m_F(\dot{x}_F^2 + \dot{y}_F^2) + \frac{1}{2}m_B(\dot{y}_B^2) + \frac{1}{2}I_T(\dot{\theta}_1)^2 + \frac{1}{2}I_F(\dot{\theta}_2)^2 \tag{4}$$

with (x_F, y_F) the coordinates of point F, y_B the y-coordinate of point B ($x_B = 0$ always), θ_1 the angle between the x-axis and segment OK and θ_2 the angle between the x-axis and segment KB.

The total potential energy of the system is given by:

$$U = m_B g(y_B - y_B^*) + m_F g(y_F - y_F^*) + m_T g(y_T - y_T^*) \tag{5}$$

with y_B^*, y_F^* and y_T^* the initial y-coordinates of respectively point B, point F and point T. Initially, the system is in static equilibrium, therefore the initial values y_B^*, y_F^* and y_T^* can be calculated by expressing horizontal, vertical and moment equilibrium in the system with an initial value for $L_C(0) = 0,42$ m. The resulting values for y_B^*, y_F^* and y_T^* are respectively 0,63 m, 0,50 m and 0,19 m.

2.2.1 Generalized forces

The generalized force Q can be defined using the concept of virtual work (D'Alembert's principle) where Q is defined as:

$$Q = \sum_{i=1}^{n} \mathbf{F}_i \frac{\partial \mathbf{r}_i}{\partial q} \tag{6}$$

with $\mathbf{F}_i$ the vector representation of the applied forces to the system and $\mathbf{r}_i$ the position vector of the place where the forces are applied.

In this paper we will focus on obtaining a realistic nonlinear model for the flexing motion of the knee (bending of the knee). In future research a similar model will be developed for the upward motion. Both motions differ in the applied forces (different directions). In the current system, we have two types of applied forces: the force in the cable and the friction forces.

The force in the cable is modeled as a spring force. The spring constant can be calculated from the force in the cable at time $t = 0$ and $t = 5$ (deepest point is reached after 5 seconds). At both times, we calculate from the static equilibrium the force in the cable from which we calculate the spring constant k as 75296 N/m and the unloaded length of the spring ℓ as $0,40$ m. The amplitude of the applied force is then $F_c = k(L_C - \ell)$ with L_C the length between point C and the rotation point of the linear actuator at current time t. The force vector $\mathbf{F}_1$ can be split in a force along the x-axis and a force along the y-axis: $\mathbf{F}_1 = F_c \sin(\theta)\ \mathbf{e}_x + F_c \cos(\theta)\ \mathbf{e}_y$. The place vector $\mathbf{r}_1$ is then: $\mathbf{r}_1 = x_C\ \mathbf{e}_x + y_C\ \mathbf{e}_y$ with x_C and y_C the coordinates of point C (in function of θ).

L_C is the distance between the knee joint axis K and the rotation point of the linear actuator. Therefore the relationship between the extending velocity $v(t)$ of the linear actuator and L_C can be expressed as $\frac{dL_C}{dt} = v(t)$ with the initial condition $L_C(0) = 0,42$ m.

There are two types of friction forces present in the current system: friction at the linear guide system that guides the body weight in a vertical direction and friction in the three revolute joints present in the system.

The friction force in the linear guide is modeled as a kinetic friction force with a kinetic friction coefficient of $\mu_k = 0,22$ (steel on steel) [17]. The friction force is then taken to be the multiplication of the friction coefficient and the sum of all horizontal forces in the system i.e. the normal force exerted on the sliding surface. This force is applied in point B resulting in a place vector $\mathbf{r}_2 = x_B\ \mathbf{e}_x + y_B\ \mathbf{e}_y$.

To model the friction forces in the revolute joints, we use again the concept of virtual work. The following relationship is valid for revolute joints:

$$\delta W = F_f \delta s = F_f r_j \delta\alpha = \tau\delta\alpha = -c_j \dot{\alpha} \delta\alpha \tag{7}$$

with δW the virtual work, F_f the friction force, r_j the radius of the joint, τ the moment around the joint, c_j the frictional damping coefficient and $\delta\alpha$ an angular virtual displacement. From this we can get the generalized force for each revolute joint (ankle, knee and hip). However, determining the values of c_j is impossible without measurements and therefore, the values where taken so that the damping of the mathematical model is similar to the damping of the real life machine. The resulting values for the frictional damping coefficient for the ankle, knee and hip joints are respectively: $c_A = 169,8$ Nms/rad, $c_K = 283$ Nms/rad and $c_H = 566$ Nms/rad.

As the system has only one degree of freedom, all coordinates and angles in the above equations can be written in function of θ resulting in an equation of motion with only one variable.

3 Simulation of the nonlinear equation of motion

The equation of motion resulting from the Lagrange theory described in previous section, can be expressed as $\ddot{\theta} = f(\dot{\theta}, \theta, L_C)$ with f representing a nonlinear function. The equation of motion will be simulated in MATLAB/SIMULINK as verification of the model. The SIMULINK scheme used for the simulation is shown in figure 2. The input signal $v(t)$ is generated by a combination of a sine wave block and a saturation block. The resulting signal is shown in figure 3a. The fact that the linear actuator can not go from complete standstill to 0.01 m/s is taken into account in the design of the input velocity signal by the oblique edges in the block wave.

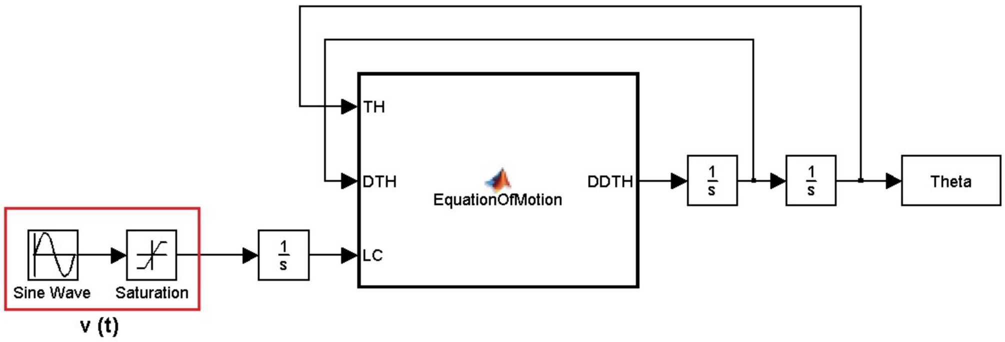
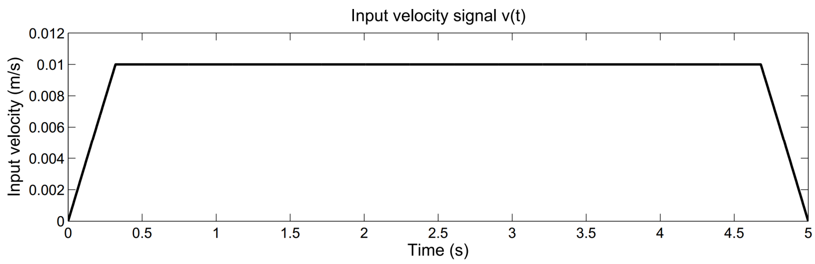
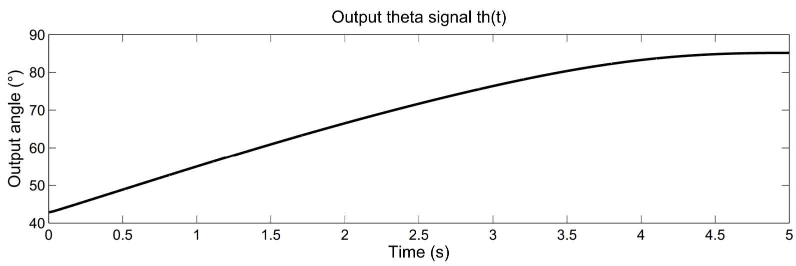


Fig. 2 Simulink scheme used for the simulation.



(a) Input velocity signal i.e. action of the linear actuator.



(b) Output angle signal i.e. the hip angle θ increases as the knee flexes.

Fig. 3 Signals from the Simulink simulations.

By using an integrator block, the signal $L_C(t)$ can be obtained from the signal $v(t)$ and the signal $L_C(t)$ is then used as an input in the nonlinear equation of motion. The resulting plot for the angle $\theta(t)$ is shown in figure 3b. The resulting signal for $\theta(t)$ increases in 5 seconds from an initial value of 43° to an end value of 85° which corresponds to physical insight into the system and to the real-life behavior of the system. We can conclude that the obtained equation of motion is a good model for the flexing motion of the knee rig.

4 Conclusion

This paper has shown that Lagrangian theory can be used to obtain an accurate equation of motion (model) for the flexing motion of a nonlinear dynamic emulator. This model is the first step towards obtaining a full model and controlling the flexion angle of the knee rig. Future work will focus on accurately modeling the extending motion of the system and the development of an adequate control strategy. Current efforts include the realization of a lab-scale setup to validate the proposed model.

References

[1] Van Haver, A., De Roo, K., Claessens, T., De Beule, M., Verdonk, P. and De Baets, P. (2013) Pilot validation study on a quasi-static weight-bearing knee rig, *Proceedings of the institute of mechanical engineers part H-journal of engineering in Medicine*, **227**(H3), 229-233.

[2] Zavatsky, A.B. (1997), A Kinematic-freedom Analysis of a flexed-knee-stance testing rig, *Journal of Biomechanics*, **30**(3), 277-280.

[3] Luyckx, T. Didden, K., Vandenneucker, H., Labey, L., Innocenti, B. and Bellemans, J. (2009) Is there a biomechanical explanation for anterior knee pain in patients with patella alta? Influence of patellar height on patellofemoral contact force, contact area and contact pressure, *Journal of bone and joint surgery- British Volume*, **91B**(3), 344-350.

[4] Miller, R.K., Goodfellow, J.W., Murray, D.W. and O'Connor, J.J. (1998) In vitro measurement of patellofemoral force after three types of knee replacement, *Journal of bone and joint surgery- British Volume*, **80B**(5), 900-906.

[5] Shoemaker, S.C., Adams, D., Daniel, D.M. and Woo, S.L. (1993) Quadriceps Anterior Cruciate Graft Interaction - An in-vitro study of joint kinematics and anterior cruciate ligament graft tension, *Clinical Orthopaedics and related research*, **294**, 379-390.

[6] De Loës, M., Dahlstedt, L.J. and Thomée, R. (2000) A 7-year study on risks and costs of knee injuries in male and female youth participants in 12 sports, *Scandinavian Journal of Medicine & Science in Sports*, **10**(2), 90-97.

[7] Cram, P., Lu, X., Kates, S.L., Singh, J.A., Li, Y. and Wolf, B.R. (2012), Total Knee Arthroplasty Volume, Utilization, and Outcomes Among Medicare Beneficiaries, 1991-2010, *The Journal of the American Medical Association*, **308**(12), 1227-1236.

[8] Centers for Disease Control National Center for Health Statistics FastStats: Inpatient Surgery. http://www.cdc.gov/nchs/fastats/insurg.htm Accessed June 25, 2013.

[9] Kurtz, S., Mowat, F., Ong, K., Chan, N., Lau, E. and Halpern, M. (2005) Prevalence of primary and revision total hip and knee arthroplasty in the United States from 1990 through 2002, *Journal of Bone and Joint Surgery - American Volume*, **87**(7), 1487-1497.

[10] Murphy, L., Schwartz, T.A., Helmick, C.G., Renner, J.B., Tudor, G., Koch, G., Dragomir, A., Kalsbeek, W.D., Luta, G. and Jordan, J.M. (2008), Lifetime Risk of Symptomatic Knee Osteoarthritis, *Arthritis & Rheumatism (Arthritis Care & Research*, **59**(9), 1207-1213.

[11] The Organisation for Economic Co-operation and Development. http://www.oecd-ilibrary.org Accessed July 10, 2013.

[12] Walker, P.S., Blunn, G.W., Broome, D.R., Perry, J., Watkins, A., Sathasivam, S., Dewar, M.E. and Paul, J.P. (1997) A knee simulating machine for performance evaluation of total knee replacements, *Journal of Biomechanics*, **30**(1), 83-89.

[13] Varadarajan, K.M., Harry, R.E., Johnson, T. and Li, G. (2009) Can in vitro systems capture the char-

acteristic differences between the flexion-extension kinematics of the healthy and TKA knee?, *Medical Engineering & Physics*, **31**, 899-906.

[14] Kirsch, L., Wirth, C.J., Kohn, D. and Glowik, A. (1998) A dynamic knee simulator with feedback control, *Journal of Biomechanics*, **31**, 143.

[15] Winter, D.A., (2009), *Biomechanics and Control of Human Movement*, John Wiley & Sons, Inc., Hoboken, New Jersey, 4th edition, 82-95.

[16] Luyckx, L., Loccufier, M. and Noldus, E. (2004) Bounded nonlinear control of a rotating pendulum system, *Computing Anticipatory Systems*, **718**, 593-600.

[17] Typical Coefficient of Friction Values for Common Materials. http://blog.mechguru.com/machine-design/typical-coefficient-of-friction-values-for-common-materials/ Accessed July 9, 2013.